

ABSTRACTS

EXPONENT FOR HEAT FLUX IN EXPERIMENTS ON HEAT EXCHANGE IN A BOILING LIQUID

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UDC 536.24: 536.25: 536.423.1

In experiments on heat exchange in boiling under conditions of free convection there holds a power law of the form

$$\alpha = Aq^n,$$

where n takes a value from 0.6 to 0.8, and, according to the data of some studies, from 0.55 to 0.95.

A comparison of a large number of published studies shows that the variation in the exponent over such wide limits is connected with a substantial indeterminacy in conducting the experiments. The exponent in (1) and the coefficient of heat exchange during boiling, other conditions being equal, can depend considerably on the mechanical and chemical purity of the heat-exchange surface, and also on the chemical purity of the coolant.

Experiments on heat exchange during boiling in a large volume are conducted presently according to two methods, which differ from each other according to the means of heating the working volume. The working volume is heated by an electric heater, coiled outside, or placed in the thermostat. We conducted experiments on the same apparatus and the same experimental section while changing the method of heating the working volume. Figure 1 shows results of experiments on the heat exchange during boiling of water under atmospheric conditions.

The figure shows that for large thermal fluxes, the magnitudes of the coefficients of heat exchange in both cases are the same, and agree with the data of [1], while for small thermal fluxes the values of the measured quantities are stratified along the different curves. The exponent in experiments with a thermostatted volume equals 0.8, and according to data obtained in a nonthermostatted volume, the exponent can be chosen from 0.6 to 0.8 in different thermal-flux ranges.

The article presents similar measurements for the boiling of Freon-21 (for two values of pressure); also given are photographs made on high-speed motion-picture film, at the transition point from convection to boiling for different methods of heating the working volume, and the most probable cause of

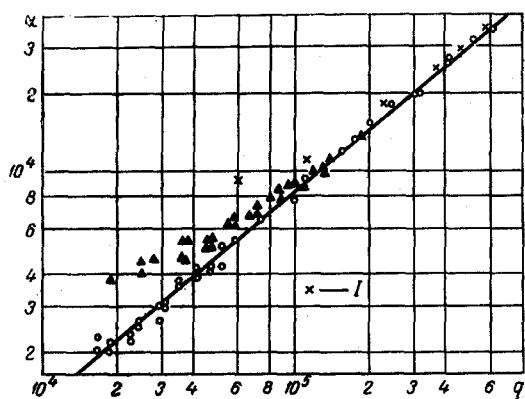


Fig. 1. Effect of method of heating the working volume on the coefficient of heat exchange during boiling of water (atmospheric pressure, the open circles indicate thermostatted volume; the solid triangles indicate nonthermostatted volume; 1) indicates the data of [1]). α , $W/m^2 \cdot \text{deg}$; q , W/m^2 .

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intensification of the heat exchange is indicated for experiments in which the working volume is heated by an electric heater. On the basis of the present experiments it is concluded that part of the work on heat exchange during boiling under conditions of free convection has been carried out incorrectly, since in these works the conditions of free convection have not been fulfilled rigorously.

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USE OF CRYOGENIC EVACUATION IN LOW DENSITY GAS-DYNAMIC APPARATUS

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UDC 536.423.4

Condensation of carbon dioxide in a circulating system onto a flat plate cooled to a temperature below the triple point, as well as the delivery characteristics of a vacuum condensation apparatus designed for gas-dynamic studies.

In the first case a jet of CO_2 escaping from a sonic nozzle strikes a cold copper disk 10 cm in diameter situated in a vacuum chamber 26 cm in diameter and 40 cm long, where it condenses into the solid phase.

The discharge rate of the carbon dioxide is measured with a flow meter, and the pressure with a thermocouple and an ionization pressure gauge.

At a pressure measuring $1 \cdot 10^{-2}$ mm Hg the specific condensation rate reaches 12.4 liter/cm² sec.

Another series of experiments was conducted on a vacuum apparatus of 3 m³ volume. Cryogenic controls are located along the chamber walls: cold shielding on one side of the chamber walls, and the working circuit and cold barrier on the side of the gas stream. Liquid nitrogen is drawn from a 1200 liter TRZhK-2u tank into the circuit tubing and then emerges into the atmosphere in a gas-vapor mixture, cooling the circuit to a temperature of $90 \pm 5^\circ\text{K}$.

A BN-4500 pump is connected to the chamber as well as a vacuum assembly for preliminary evacuation and evacuation of noncondensing gases.

The limiting delivery characteristics were obtained for the apparatus studied during condensation of carbon dioxide in the pressure range of $3 \cdot 10^{-5}$ - $2 \cdot 10^4$ mm Hg with cooling of all surfaces with liquid nitrogen. In calculating the delivery volume the results were presented in terms of the chamber pressure and room temperature. At a pressure on the order of $1 \cdot 10^{-4}$ mm Hg the output of the apparatus exceeded $2 \cdot 10^6$ liter/sec.

In the chamber pressure range of $3 \cdot 10^{-5}$ - $1 \cdot 10^{-1}$ mm Hg the mass delivery characteristics were taken for a cryogenic pump without a "cold trap" in the conical section of the chamber. For carbon dioxide delivery rates above 18 g/sec ($P > 4 \cdot 10^{-3}$ mm Hg) a sharp increase in the chamber pressure occurs because of an increase in the temperature on the surface of the cryogenic deposit.

The apparatus has an exhaust time for the system of about 20 min with a liquid nitrogen consumption for cooling of about 50 liters.

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At high carbon dioxide delivery and at $P > 1 \cdot 10^{-2}$ mm Hg a temperature of up to 120°K was recorded at "hot spots."

A stationary state was achieved at carbon dioxide delivery rates of about 30 g/sec, which corresponds to a chamber pressure of $1 \cdot 10^{-1}$ mm Hg.

Instruments which use cryogenic evacuation are now widely used in the practice of gas-dynamic studies. The apparatus described is used for the study of hypersonic streams of low density in a wide range of pressures and flow rates.

EFFECT OF PARAMETERS ON THE BOUNDARY OF FLOW STABILITY IN A SYSTEM OF PARALLEL STEAM-GENERATOR TUBES

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V. G. Zinkevich, and R. I. Kalinin

The flow rate of a medium corresponding to the boundary of vibrational stability of a flow in a system of parallel heated tubes is a complicated function of a large number of parameters:

$$(\rho w)_{\text{bo}} = f(P, \Delta i_0, q, d, l, \xi_{\text{in}}, \xi_{\text{out}}, \theta).$$

The qualitative effect of most of them has been determined experimentally; the quantitative effect, however, of the individual parameters and the mechanism of their action have not been established for the general case.

The article gives results of investigations of the quantitative effect of the individual parameters on the boundary of stability of the flow, based on a direct numerical digital-computer integration of the system of partial differential equations that describes the process.

In the range of variation of parameters being considered, the change in specific thermal loading (q) and the heated length of tube (l) causes, other conditions being equal, an almost directly proportional change, and for a change in internal diameter, an almost inversely proportional change, in the limiting mass flow rate; i.e., the relation between the lengths of the evaporative section and the economizer section remains almost constant. The deviation from the proportional dependence in the parameter range being considered does not exceed 10%, and is explained by a certain change in the steam pressure and the values of the ratio of the coefficients of friction in real and homogeneous two-phase flow with change in the Reynolds number Re and the inertia of the coolant.

The variation in the thermal loading along the tube length somewhat shifts the stability boundary. The article indicates the limiting values of the variation below which the displacement of the limiting flow rate does not exceed 10% and the limiting flow rate can be determined according to the thermal flux averaged over the length of the tube.

A change in the tube length affects the nature of the pulsations. This effect for the same final steam-content weight is expressed with an increase in the tube length: first, in an increase of the pulsation period that is almost proportional to the increase in the length being heated, and secondly, in the slower increase in pulsation amplitude with decrease in flow rate below the limiting value. The paper discusses the mechanism of this effect, explained by an increase in the inertia of the medium, which inhibits the growth of pulsations.

The presence of a preliminary nonheated section of tube at the inlet increases the flow stability. Such a section is more effective than a throttling that is equivalent to it in drag at the inlet owing to the slower rise in pulsation amplitude for a decrease in flow rate below the limiting value. The mechanism of this effect is discussed in the article by means of an analysis of the momentum equation. For calculations of the limiting flow rate we can, making the proper allowance, assume a preliminary section having a concentrated drag.

An investigation of the effect of throttling at the inlet (ξ_{in}) showed that the limiting flow rate, other conditions being equal, depends on the throttling hyperbolically, i.e., with increasing throttling, its effectiveness falls. This is explained by the fact that a decrease in the limiting flow rate as a result of the throttling leads to the occurrence at the end of the tube of a section with small intensity of change of flow rate along the tube length (i.e., a section with large weight steam-content, and with further decrease of flow rate – with superheating), which is like throttling at the output, and decreases the flow stability. Practically, the boundary that has the greatest effect on the inlet throttling is $\xi_{in} = 500$.

The effect of underheating of the medium at the inlet (Δi_0) on the boundary of flow stability appears ambiguously in various intervals of its variation. For the range of parameters under consideration, an increase in underheating beginning with 15–25 kcal/kg, increases the flow stability; increase of underheating from 0 to 15–25 kcal/kg decreases it. The article gives a detailed explanation of the mechanism of such an effect of underheating on the stability boundary, related to the predominance in various intervals of a factor for the increase in length of the evaporative section or decrease in the magnitude of oscillations of the boundary of the economizer section; the first worsens the flow stability, and the second stabilizes it.

The article presents the quantitative dependencies based on the effect of pressure on the flow stability, and the mechanism of this effect is described.

All the enumerated laws on the effect of the parameters on the flow stability boundary are completely confirmed by the experimental data on the authors and other investigators, and presented in the form of graphs and tables.

DETERMINATION OF THE TEMPERATURE AND THE RADIATION CHARACTERISTICS OF FLAMES WITH THE TECHNIQUE OF BEAM-LENGTH VARIATION

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Experimental data on the temperature distribution and the radiation characteristics (ray attenuation coefficient and degree of blackness) as functions of the volume are required for research on heat and mass exchange processes occurring in flames and furnaces when various fuels are burnt. Due to the aggressive interaction between the combustion products and the thermal sensor, contact measurements of the flame temperature are often impossible. It is therefore proposed to use a contactless technique of changing the length of the beam as a generalization of the known method of doubling the beam length. The proposed method comprises layerwise measurements of the radiant heat fluxes originating from the flame for various lengths of the beam on the background of a model of a cold, absolutely black body; the measurements are made with a narrow-angle radiometer and thereafter used in calculations of temperature fluxes and the radiation characteristics. The instruments must have the form of water-cooled movable probes.

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Large systematic and functional errors can be made. The first type of errors results from aerodynamic and thermal distortions of the parameters of flame sections near the boundary with the cold surface of the probe. The functional errors result from the complicated dependence of the parameters to be determined upon the determining parameters, so that a small error which is made in measurements of the radiant flux results in large errors of the temperature determination and the determination of the radiant characteristics with formulas.

In order to completely eliminate methodological errors, a measurement scheme is proposed and equations for evaluating the results of the measurements are derived. Also developed is a technique of graphically averaging the results of heat flux measurements for various lengths of the beam; this technique makes it possible to reduce the functional errors by one order of magnitude. It is possible to estimate the magnitude of the functional errors in direct calculations which are based on the equations when the graphical averaging of the heat flux is used; the estimates are made according to the resulting analytic dependencies and graphs.

The technique of determining the temperature and the radiation characteristics was tested in the chamber of a furnace. The combustion products of natural gas were the objects of the investigation in which the length of the beam was varied between 0 and 0.8 m. The results of the experiments are listed in Table 1.

THE TEMPERATURE FIELD OF ROCKS, TAKING INTO ACCOUNT THE INTERACTION OF BOREHOLES

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UDC 546.12

The combined use of two or more parallel boreholes is planned in some systems which make use of the heat of the Earth's interior. In connection to this work, the temperature distribution around two interacting cylindrical boreholes is considered when the wall temperature of the boreholes is assumed to be a given function of time. The two-dimensional case is considered, i.e., the axes of the boreholes are assumed to be parallel and the heat flux along the axes of the boreholes is assumed to be negligibly small in the rocks, compared to the radial heat flux.

TABLE 1. Results of Tests on the Determination of T and k

No. of sample	Parameters	Test 1, $\alpha = 1,0$	Test 2, $\alpha = 0,7$	Parameters	Test 1 $\alpha = 1,0$	Test 2 $\alpha = 0,7$
1	T , °K	1630	1648	k , 1/m	0,287	0,205
2	T' , °K	1303	1878	k' , 1/m	0,140	0,092
3	$\delta T'$, %	20	14	$\delta k'$, %	51	55
4	T'' , °K	1643	—	k'' , 1/m	0,316	—
5	T''_p , °K	1653	—	k''_p , 1/m	0,213	—
6	T''_p , °K	1648	1671	k'' , 1/m	0,264	0,190
7	$\delta T''$, %	1,1	1,4	$\delta k''$, %	8,0	7,3

Note. 1) true value; 2) directly calculated with formulas; 3) error made in the determinations based on direct calculations; 4, 5, 6) values obtained from graphical analysis; 7) error of graphical analysis; T denotes the temperature of radiating gases; k denotes the ray-attenuation coefficient.

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A solution to the nonstationary equation of heat conduction was obtained with the operator method and by superposition of fictitious temperature at the borehole walls. The solution of our work is an approximation and is valid whenever the distance between the axes of the boreholes is much greater than their radii. The reduction of the temperature image with the aid of a Laplace transform was made in two ways for the particular case in which the temperature at the borehole walls is equal and constant.

In the first way, an expression for the temperature was obtained in the form of an infinite series with terms which are single integrals of rather complicated expressions including Bessel functions.

In the second way, the expression for the temperature has also the form of a series in which the multiplicity of the proper integrals appearing in the terms of the series increases with increasing number of the terms. However, the functions in the expressions under the integral sign are tabulated functions which facilitates the calculations. The first three terms of the series have the form

$$t/T_0 \approx f(R, Fo) + f(R', Fo) - \int_0^{Fo} f'_{Fo}(R, \eta) f(R', Fo - \eta) d\eta, \quad (1)$$

where t denotes the temperature of the rock; T_0 denotes the temperature of the borehole wall; R and R' denote the dimensionless distances to the axes of the first and second borehole, respectively; F_0 denotes the Fourier integral; and $f'_{Fo}(R, \eta) = [\partial f(R, Fo)/\partial Fo]_{Fo = \eta}$. The function $f(R, Fo)$ is tabulated and renders the rock temperature in dependence of a single, independently operating borehole.

It follows from Eq. (1) that the first two terms are the sum of the temperature generated by two independently working boreholes, whereas the third term is a correction which accounts for the interaction between the boreholes. The numerical calculations show that the approximation formula (1) can be used in a large variability range of the Fourier criterion.

DETERMINATION OF THE ROOTS OF SEVERAL TRANSCENDENTAL EQUATIONS CONTAINING BESSEL FUNCTIONS OF THE FIRST AND SECOND KIND AND THEIR DERIVATIVES

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UDC 517.942.9

The determination of the roots of the following equations, which contain Bessel functions and their derivatives, is needed in the solution of several problems of engineering:

$$J_n(\chi) N'_n(t\chi) - N_n(\chi) J'_n(t\chi) = 0, \quad (1)$$

$$J'_n(\chi) N_n(t\chi) - N'_n(\chi) J_n(t\chi) = 0, \quad (2)$$

$$J_{n+1}(\chi) N_n(t\chi) - N_{n+1}(\chi) J_n(t\chi) = 0, \quad (3)$$

$$J_n(\chi) N_{n+1}(t\chi) - N_n(\chi) J_{n+1}(t\chi) = 0, \quad (4)$$

$$J_n(\chi) N_n(t\chi) - N_n(\chi) J_n(t\chi) = 0, \quad (5)$$

$$J'_n(\chi) N'_n(t\chi) - N'_n(\chi) J'_n(t\chi) = 0. \quad (6)$$

The numerical determination of the roots of these expressions is rather difficult in view of the multiplicity of the roots. By analyzing the sequence of the roots of the Bessel functions of the first and second kind and their derivatives (which are tabulated functions), we obtain the limits within which the roots of these equations can be found for any value of the parameter t . Table 1 is a list of the intervals in which the m -th root of the corresponding equation is situated, the root being denoted by $\chi_{n,m}$.

TABLE 1

Boundaries within which the roots are situated	Applicability range
$C_{p,q} \leq t\chi_{n,m} \leq D_{p,q}$ $D_{p,q} \leq t\chi_{n,m} \leq C_{p,q+1}$ $C_{p,q+1} \leq t\chi_{n,m} \leq D_{p,q+1}$ $D_{p,q+1} \leq t\chi_{n,m} \leq C_{p,q+2}$	$0 \leq 1/t \leq B_{p,1}/D_{p,q}$ $B_{p,1}/D_{p,q} \leq 1/t \leq A_{p,1}/C_{p,q+1}$ $A_{p,1}/C_{p,q+1} \leq 1/t \leq B_{p,2}/D_{p,q+1}$ $B_{p,2}/D_{p,q+1} \leq 1/t \leq A_{p,2}/C_{p,q+2}$

The coefficients A, B, C, and D for the corresponding equations have the following values:

$$A_{p,q} = j_{n,m}; B_{p,q} = y_{n,m}; C_{p,q} = j'_{n,m}; D_{p,q} = y'_{n,m} \quad (1')$$

$$A_{p,q} = y'_{n,m}; B_{p,q} = j'_{n,m}; C_{p,q} = y_{n,m}; D_{p,q} = j_{n,m} \quad (2')$$

$$A_{p,q} = j_{n+1,m}; B_{p,q} = y_{n+1,m}; C_{p,q} = j_{n,m}; D_{p,q} = y_{n,m+1} \quad (3')$$

$$A_{p,q} = j_{n,m}; B_{p,q} = y_{n,m}; C_{p,q} = j_{n+1,m-1}; D_{p,q} = y_{n+1,m} \quad (4')$$

$$A_{p,q} = j_{n,m}; B_{p,q} = y_{n,m}; C_{p,q} = j_{n,m}; D_{p,q} = y_{n,m+1} \quad (5')$$

$$A_{p,q} = y'_{n,m}; B_{p,q} = j'_{n,m}; C_{p,q} = y'_{n,m-1}; D_{p,q} = j'_{n,m} \quad (6')$$

The following notation is used to Eqs. (1)-(6) and (1'-6'): $t \geq 1$ denotes a parameter; J_n , N_n , J'_n , and N'_n denote Bessel functions of the first and second kind and their first derivatives of n -th order, respectively; n denotes some positive number (integer or fraction), including zero; $j_{n,m}$, $y_{n,m}$, $j'_{n,m}$, and $y'_{n,m}$ denote the m -th roots of the Bessel functions of the first and second kind and their derivatives, respectively.

It follows from an inspection of the table, that the m -th root of Eqs. (1)-(6), except for the first root of Eq. (6), varies between 0 and ∞ when the parameter t varies between ∞ and 1. For $m = 1$, the tabulated values corresponding to Eq. (6) degenerate to the relation

$$0 \leq \chi_{n,1} \leq j'_{n,1}/t.$$

By expanding the terms of Eq. (6) near the point $t = 1$, we obtain that the first root of Eq. (6) tends to n for $t \rightarrow 1$. Thus, we obtain the relation

$$n \geq \chi_{n,1} \leq \frac{j_{n,1}}{t}, \quad \text{if } 1 \leq t \leq \infty.$$

for the first root of Eq. (6).

ANALYTIC INVESTIGATION OF THE POSSIBILITY OF MONITORING TEMPERATURE FIELDS BY DISTRIBUTED THERMISTORS

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A group of thermistors in the form of conductors which are electrically insulated from each other and from the medium to be monitored is considered. Due to a given inhomogeneity in the distribution of the

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electrical resistance of each conductor along the line of temperature measurement (obtained by nonuniformly winding the conductor on a support or by some other means), the Fourier coefficients of the temperature distribution function can be uniquely determined from the resistances of the conductors. It is shown that when conductors with circular cross section are used, a reduction of the measurement errors can be obtained by reducing the diameter of the conductors and by increasing the heat transfer coefficient.

The article includes an example of using the information on the Fourier coefficients for optimizing the control of thermal processes.

NET SOLUTION METHOD OF NONSTATIONARY HEAT CONDUCTION PROBLEM FOR A GAS-TURBINE DISC WITH SHROUDED BLADES ON A SUPPORT

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UDC 621.438-254:536.2

A gas-turbine disc is considered with shrouded blades and of variable cross-section and also with a central hole. The heat-conduction problem for the body system disc-blades can be formulated as a system of differential equations

$$\frac{\partial t_d(r, \tau)}{\partial \tau} = \frac{\alpha_d}{r_2(r)} \cdot \frac{\partial}{\partial r} \left[r_2(r) \frac{\partial t_d(r, \tau)}{\partial r} \right] - \frac{\alpha_6(r)}{c_d \rho_d r^2} [t_d(r, \tau) - t_b(r)], \quad (1)$$

$(r_2 \leq r \leq r_1, \tau > 0),$

$$\frac{\partial t_b(x, \tau)}{\partial \tau} = \frac{\alpha_b}{S(x)} \cdot \frac{\partial}{\partial x} \left[S(x) \frac{\partial t_b(x, \tau)}{\partial x} \right] - \frac{\alpha_9(x) P(x)}{c_b \rho_b S(x)} [t_b(x, \tau) - t_b^*(x)], \quad (2)$$

$(0 \leq x \leq l, \tau > 0).$

The blade fin attains its maximum temperature, for example, in the middle; it is advisable therefore to simplify the problem to consider first of all its head portion ($0 \leq x_1 \leq l_1$). The boundary conditions can then be specified as follows:

for the disc

$$\frac{\partial t_d(r_2, \tau)}{\partial r} = 0; \quad \frac{\partial t_d(r_1, \tau)}{\partial r} = \varphi_1 t_b(0, \tau) + \varphi_2 t_d(r_1, \tau) + \varphi_3 \quad (3)$$

and for the blade

$$\frac{\partial t_b(0, \tau)}{\partial x_1} = \varphi_4 t_b(0, \tau) + \varphi_5 t_d(r_1, \tau) + \varphi_6; \quad \frac{\partial t_b(l_1, \tau)}{\partial x_1} = 0. \quad (4)$$

The constants $\varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5, \varphi_6$ can be found from the heat-balance equations for $x_1 = 0$ and $r = r_1$. Equation (2) with the boundary conditions

$$\frac{\partial t_b(0, \tau)}{\partial x_2} = 0; \quad t_b(l_2, \tau) = t_d^*(l_2) - [t_d^*(l_2) - t_0] e^{-m\tau} - [t_d^*(l_2) - t_{ss}] (1 - e^{-m\tau}). \quad (5)$$

is the starting point for the tail portion of the blade ($0 \leq x_2 \leq l_2$). In the above t_0 is the initial temperature; t_{ss} is the stationary temperature of the heated shrouded wing which is found from the heat-balance equation; $m = 0.5(\alpha_1 + \alpha_2) S_{SS} / Mc_b$ is the temperature of the regular heating of the upper shrouded wing.

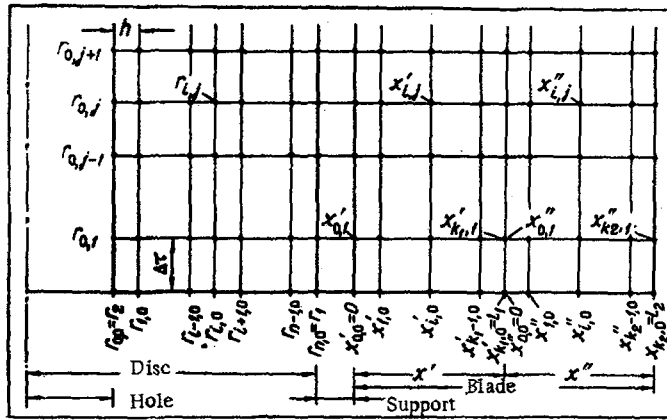


Fig. 1. Net scheme.

The network scheme for the first problem is shown in the graph below. It can be seen that the solution for the disc and for the blade is sought at the nodes of two nets which are interrelated by the boundary conditions suitably represented in a difference form.

An explicit formula was used for integration. This is possible in our case since under the stability conditions one has

$$\Delta\tau \leq \frac{h^2}{2\max S(x)} \text{ and } \Delta\tau \leq \frac{h^2}{2\max P(r)}, \quad (6)$$

and the functions $S(x)$ and $P(r) = rz(r)$ do not vary rapidly. The integration steps are selected as follows. At the beginning a step h was selected, equal for example, to $0.15 r_1$. This enabled one to bring down the step $\Delta\tau$ to 0.5 sec. It was found by trial calculations with three values of $\Delta\tau$ from the highest value downwards that the solution converges rapidly and that there is a good agreement with experimental results.

CONVECTIVE DIFFUSION IN A MEDIUM WITH STAGNANT ZONES

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UDC 532.7+66.02

The paper deals with the radial motion of the dispersion medium in a static layer of particles with stagnant zones.

A system of differential equations is presented that incorporates the dynamically less active regions and the mass transfer between these and the stagnant zones.

The Laplace transforms of the differential equations in partial derivatives are used to find an expression from which one can derive at once the solution to the initial problem from the known solution to the equation for convective diffusion. The stagnant zones are without effect for time $\tau \rightarrow \infty$, but at smaller times these zones affect the time scale and the concentration relaxation. The solution coincides exactly with that for the corresponding convective-diffusion problem in the absence of stagnant zones if there is an adequate rate of exchange between the flowing and stagnant zones, no matter what the size of the flow zone. This means physically that the stagnant zones do not appreciably influence the concentration distribution if the mass-transfer rate is high.

A relationship is given between the concentrations in the flowing and stagnant zones, which shows that the concentration beyond the stagnant zone varies exponentially when the input has a constant concentration.

AN ELECTRICAL MODEL FOR EXAMINING NONLINEAR PROBLEMS IN THERMAL CONDUCTION

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UDC 536.2.01

The following is the dimensionless form of the equations for the temperature distribution in a solid with variable temperatures in the medium, a variable heat-transfer factor, and temperature dependence of the thermophysical properties:

$$\frac{\partial}{\partial x} \left[\bar{\lambda}(\theta) \frac{\partial \theta}{\partial x} \right] + \frac{\partial}{\partial y} \left[\bar{\lambda}(\theta) \frac{\partial \theta}{\partial y} \right] + \frac{\partial}{\partial z} \left[\bar{\lambda}(\theta) \frac{\partial \theta}{\partial z} \right] = \bar{c}(\theta) \bar{\gamma}(\theta) \frac{\partial \theta}{\partial Fo} \pm Po, \quad (1)$$

$$Bi(Fo) [\theta_c(Fo) - \theta_b] = -\bar{\lambda}(\theta) \frac{\partial \theta}{\partial N}, \quad (2)$$

$$\theta(\bar{x}, \bar{y}, \bar{z})_{Fo=0} = f(\bar{x}, \bar{y}, \bar{z}). \quad (3)$$

The integral

$$\Phi = \int_0^{\theta} \bar{\lambda}(\theta) d\theta \quad (4)$$

is used to convert (1) to

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} = \frac{1}{a(\theta)} \frac{\partial \Phi}{\partial Fo} \pm Po(\theta), \quad (5)$$

where

$$\bar{a}(\theta) = \frac{\bar{\lambda}(\theta)}{c(\theta) \bar{\gamma}(\theta)}.$$

The left side of (5) then becomes linear, so one can readily represent it with a passive analog, e.g., an R network. Only Liebman's method can be used to represent the right side. Models with continuous solution (RC networks) are unadapted to this.

The basic component in the device is an RNR network, which consists of an R network with nonlinear electronic units connected to its nodes.

A nonlinear unit includes a functional unit, a multiplier, a differentiator, and a controlled current stabilizer; it receives the node voltage and produces a current proportional to the right side of (5), which is passed to a node in the R grid.

Boundary conditions of the third kind are provided by a device for specifying nonlinear time-varying boundary conditions.

Condition (2) remains nonlinear after the transformation of (4):

$$Bi(Fo) [\theta_c(Fo) - \theta_b(\Phi)] = -\frac{\partial \Phi}{\partial N} \quad (6)$$

and cannot be realized with the methods and devices widely used in electrical simulation of physical fields. The above device incorporates not only the nonlinearity $\theta_b = f(\Phi)$ in (6) but also the Fo dependence of θ_c and Bi. It consists of two functional shapers FS₁ and FS₂, a multiplier, a functional converter FC, and a controlled current stabilizer CCS.

The potential at the boundary point corresponding to Φ_b is passed to the input of FC, where it is converted to a voltage proportional to Φ_b , which is added to the voltage produced by FS₂ and passes to one input of the multiplier. The second input receives from FS₁, a voltage proportional to the time variation in Bi. The output voltage is proportional to the left side of (6) and is passed to the CCS, which transforms it proportionately to a current, which is passed to the boundary point, which corresponds to realization of the nonlinear boundary condition of (6).

These devices provide completely general continuous solution of nonlinear problems in field theory, which was previously impracticable with devices providing continuous solution.

THERMAL CALCULATION OF CURRENT LEADS IN CRYOGENIC ELECTROTECHNICAL EQUIPMENT

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In the article the optimum geometrical characteristics are determined for the current leads in electrotechnical equipment cooled by liquid helium, which assure minimum heat flow to the liquid. It is assumed that complete heat exchange takes place between the coolant vapors and the surface of the current lead (temperatures of the lead and vapor are identical in any cross section), while temperature changes in the physical properties of the lead material (thermal conductivity, electrical resistance) follow the Wiedmann-Franz law.

With these assumptions an analytical solution is obtained for stationary heat flow at the one-dimensional level. The expression for determination of the minimum volatility has the form

$$\frac{1}{2} \ln | 4(A - A_0)^2 k^2 - 4(A - A_0)Ak + 1 | + \frac{1}{\sqrt{\frac{1}{A^2} - 1}} \left[\operatorname{arctg} \frac{2\left(1 - \frac{A_0}{A}\right)k - 1}{\sqrt{\frac{1}{A^2} - 1}} + \operatorname{arctg} \frac{1}{\sqrt{\frac{1}{A^2} - 1}} \right] = \ln \frac{T_H}{T_e}, \quad (1)$$

where $A = mc_p/2I\sqrt{L}$; $A_0 = m_0c_p/2I\sqrt{L}$; $k = r/T_e c_p$ are dimensionless parameters; m is the coolant volatility (at one lead) determined by the total heat flow; m_0 is the volatility determined by the heat flow along the leads; c_p is the heat capacity of the coolant vapors; r is the heat of vaporization of the coolant; I is the working current; L is the constant of the Wiedemann-Franz law; T_H and T_e are the temperatures of the hot and cold ends of the current lead.

To determine the minimum dimensionless volatility (A) the dependence (1) should be used, if at fixed T_e , T_H , and k the given minimum value of A_0 can be evaluated according to the equation

$$\frac{1}{2} \ln | 4(1 - A_0)^2 k^2 - 4(1 - A_0)k + 1 | - \frac{2(1 - A_0)k}{2(1 - A_0)k - 1} = \ln \frac{T_H}{T_e}. \quad (2)$$

Otherwise the dependence derived in the article can be used.

The results of calculations for current leads (Al 99.99%) with liquid helium as the coolant ($T_e = 4.2^\circ\text{K}$; $k = 0.916$) are presented in the form of the dependence of the minimum volatility (A) and optimum geometrical parameter $\Pi\sqrt{L}/s$ on the temperature at the hot end of the lead (l and s are the length and cross-sectional area of the lead). It is shown that the volatility and geometrical parameter have a slight dependence on the temperature of the hot end in the interval of 20 - 300°K . A change in the parameter A_0 within the limits of 0 - 0.4 leads to a negligible increase in the volume of helium evaporated.

TRANSIENT THERMAL CONDUCTION BY COAXIAL INSULATED DISCS

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The discs are mounted on a central thin-walled tube of high conductivity, and this tube is at the same temperature at all points. The discs vary in thickness and extend radially to infinity; they differ in thermophysical parameters. A constant total heat flux is supplied to the tube from the start, and this is taken up completely by the discs.

The problem is solved via Laplace transformation with respect to dimensionless time; general solutions are derived for the temperature distribution in the system and the heat flux at the internal surface of each disc, with corresponding solutions for small Fo .

Results are given for a system of two discs (ceramic and glass) for small Fo .

The results are also solutions for the pressure distribution in a system of isolated strata (or one stratum with isolating bands) when a liquid is pumped in through a central borehole.